# Perceptive Kuramoto Oscillators - PeKO

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# Synchrony is a natural phenomenon

# Outline

# Recap: Oscillator Network

- Previous talk: Perceptual Grouping with Oscillators

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- Frequency update:

$$\omega_m = \omega_0 \cdot \underset{\alpha}{\text{argmax}} \Big( \sum_{n \in \mathcal{N}(\alpha)} \mathbf{F}_{mn} \cdot \frac{1}{2} (cos(\theta_n - \theta_m) + 1) \Big)$$

# Recap: Evaluation

- Comparison to the CLM, similar settings for both
- IA matrix with 1000 features in ten groups
- 100 layers, 100 discrete frequencies
- All with different amounts of noise in the IA matrices
- 500 trials for each condition

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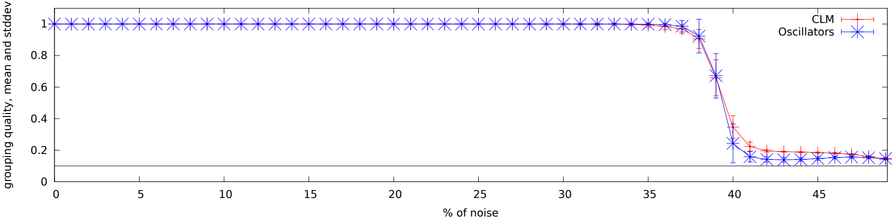
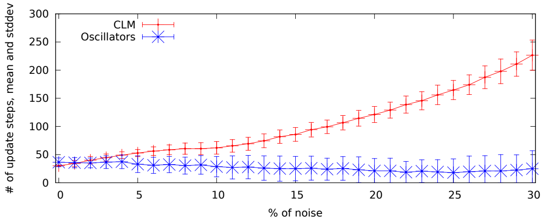
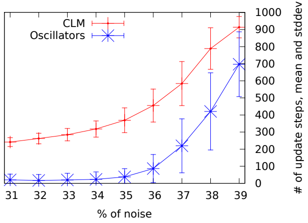


# Recap: Evaluation Results

- Previous talk: Evaluation results
- Evaluation revealed:
  - Quality comparable to the CLM
  - Computational complexity reduced
  - Grouping speed increased



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- Both models converge for 500 steps
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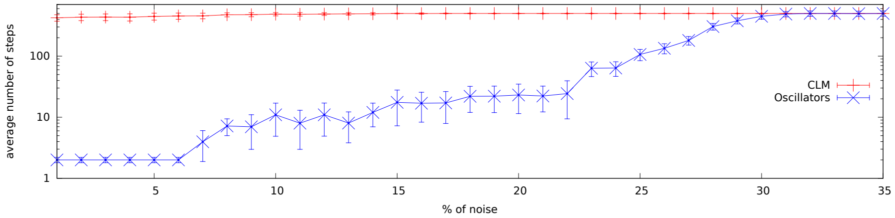
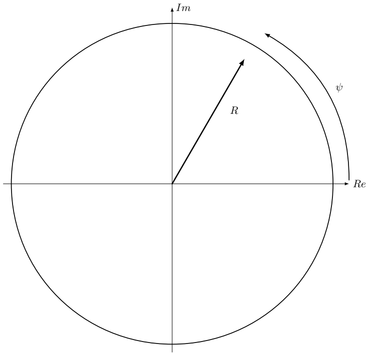


# Stablility

- PeKO achieves good grouping results.
- How to assess the grouping quality?

# Kuramoto Order Parameter

- Complex parameter, $r$ and $\psi$
- $re^{i\psi} = \frac{1}{N}\sum_{n=1}^{N} e^{i\theta_n}$
- $r$ is phase coherence $\in [0, 1]$
- $\psi$ is average phase



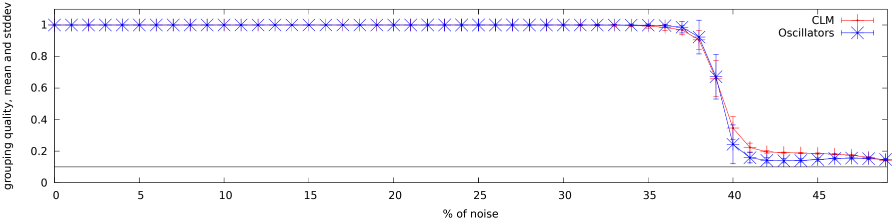

complex plane

# Order Parameter

- Needs to be adapted for discrete frequencies:

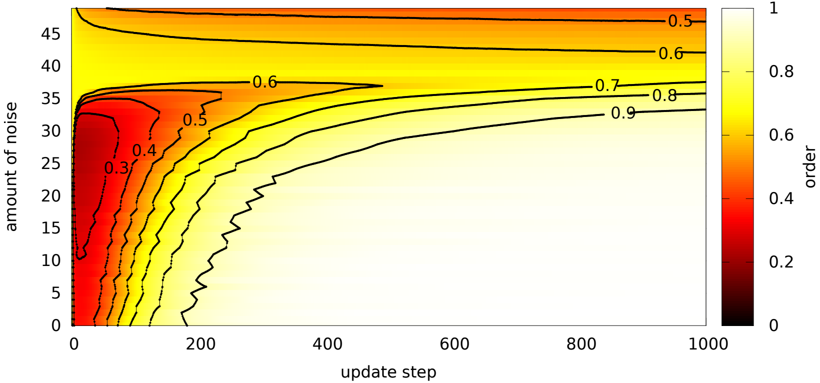
$$r_\alpha e^{i\psi_\alpha} = \frac{1}{\mathcal{N}_\alpha} \sum_{n=1}^{\mathcal{N}_\alpha} e^{i\theta_n} \quad \textbf{if} \quad \mathcal{N}_\alpha \neq 0$$

# Order Behavior



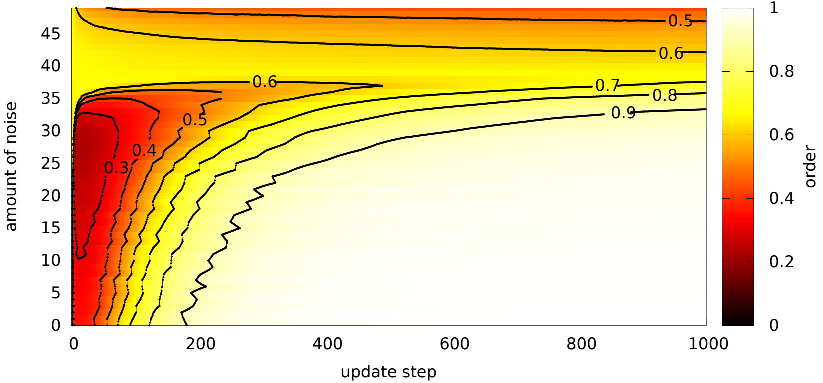

- Recap: Grouping quality with respect to noise

# Order Behavior




- Order parameter $\bar{r}$ wrt. noise and time

# Order Behavior




- Order parameter can be used to assess grouping quality

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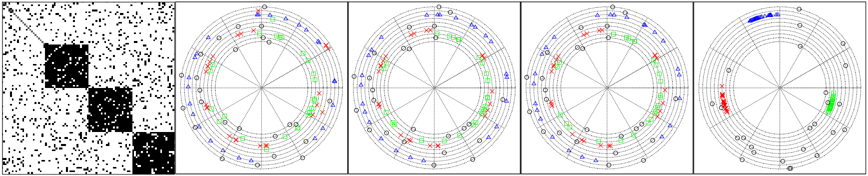


(f) IA Matrix. (g) Step 1. (h) Step 2. (i) Step 3. (j) Step 50.

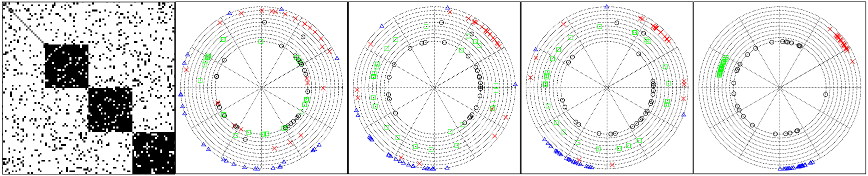
# Dealing with spurious features

- Background Layer: Nice idea "borrowed" from the CLM
- Introduced as special frequency
- Possesses "chaotic" coupling:

$$\dot{\theta}_m = \omega_m + Kr\sin(\psi - \theta_m).$$

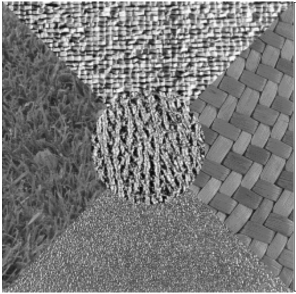
# Dealing with spurious features

- Same example as before
- Spurious features are collected by the background frequency

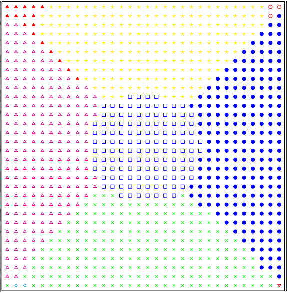


(k) Initialization. (l) Step 1. (m) Step 2. (n) Step 3. (o) Step 50.

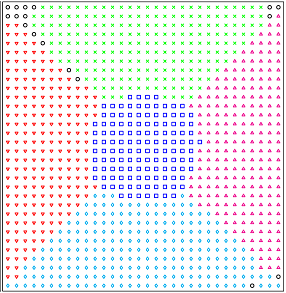
# Real World Example: Texture Grouping



(p) Input image.



(q) Without background.



(r) With background.

# Improved Learning of Lateral Interactions

- Idea from S. Weng
- Represent feature compatibility by distance functions
- Create prototypes with VQ
- Labeled examples are used to decide if $+/-$ interaction

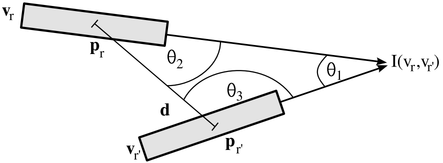
# Improved Learning of Lateral Interactions

- Original approach used Activity Equilibrium VQ
- Replaced by ITVQ
  - Better distribution of prototypes
- Evaluated in contour grouping task

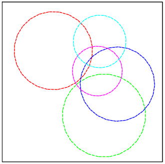
# Improved Learning of Lateral Interactions

- Evaluated with three kinds of shapes
- 200 trials for each shape
- Four conditions
  - CLM with AEV
  - PeKO with AEV
  - CLM with ITVQ
  - PeKO with ITVQ

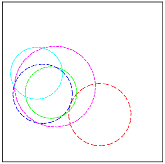
# Feature and Data Example




(s) Oriented edge features.

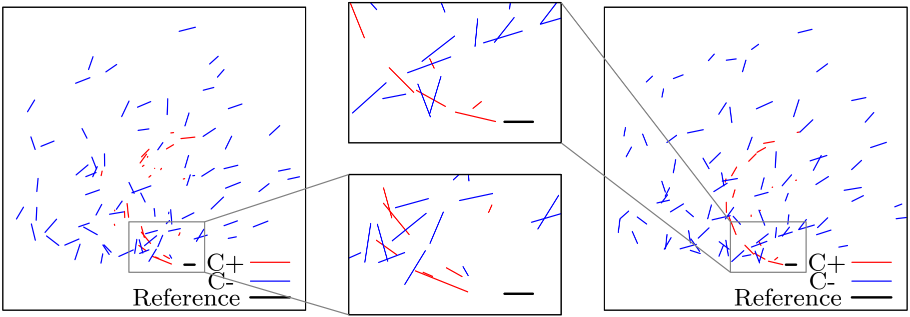


(t) “Easy” problem.



(u) “Hard” problem.

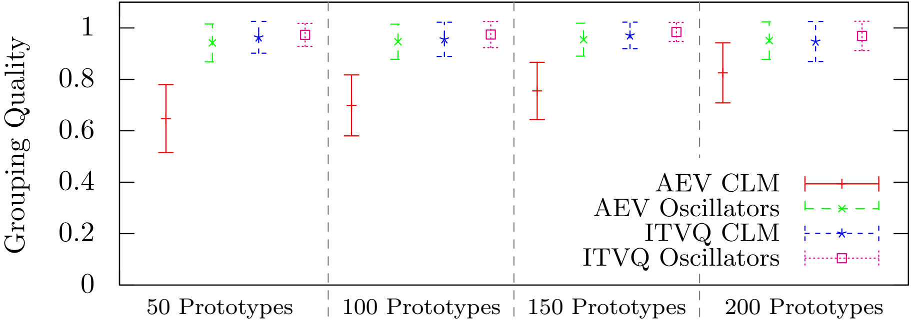
# Example of generated Prototypes




ITVQ Prototypes

AEV Prototypes

# Improved Learning of Lateral Interactions - Results



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- Order allows assessment of grouping quality
- “Chaotic” frequency handles spurious features
- Learing of lateral interactions is improved

# Thank You!

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## Any Questions?

# ITVQ Update Rule

- Minimize Cauchy-Schwartz Divergence
- $x_0$ is input, $x$ prototypes
- Fixed point update rule:

$$x_i^{t+1} = \frac{\sum_{j=1}^{N_0} G_\sigma(x_i^t - x_{0j}) x_{0j}}{\sum_{j=1}^{N_0} G_\sigma(x_i^t - x_{0j})} - c \frac{\sum_{j=1}^{N} G_\sigma(x_i^t - x_j^t) x_j^t}{\sum_{j=1}^{N_0} G_\sigma(x_i^t - x_{0j})} + c \frac{\sum_{j=1}^{N} G_\sigma(x_i^t - x_j^t)}{\sum_{j=1}^{N_0} G_\sigma(x_i^t - x_{0j})} x_i^t; \quad c = \frac{N_0}{N} \frac{V(X, X_0)}{V(X)}$$

# Learning of Lateral Interactions